TOPOLOGY IV - MID-SEMESTRAL EXAM

Time : 2 hours Max. Marks : 40

Answer all questions. You may use results proved in class after correctly quoting them.

(1) Let V be a 4-dimensional vector space over $\mathbb{R}$ and $\{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4\}$ a basis of $\mathrm{Alt}^1(V)$. Let $A = (a_{ij})$ be a skew-symmetric matrix and define

$$\alpha = \sum_{i<j} a_{ij}\, \epsilon_i \wedge \epsilon_j.$$

Show that $\alpha \wedge \alpha = 0$ if and only if $\det(A) = 0$. If

$$\alpha \wedge \alpha = \lambda \cdot \epsilon_1 \wedge \epsilon_2 \wedge \epsilon_3 \wedge \epsilon_4,$$

then what is the connection between λ and $\det(A)$. [4+4+2]

(2) Show that the 1-form

$$\omega = \frac{-y}{x^2+y^2}\, dx + \frac{x}{x^2+y^2}\, dy$$

on $\mathbb{R}^2 - 0$ is closed but not exact. [4+6]

(3) Compute $H^p(\mathbb{R}^2 - 0)$. [10]

(4) For $n \geq 2$, show that there does not exist a continuous map $f : D^n \longrightarrow S^{n-1}$ with $f/S^{n-1} = id$. Use this to prove the Brouwer's fixed point theorem. Give complete details. [6+4]